Module-II

**Polarization**

**Introduction:**

Interference and diffraction of light, which we have studied in module-I, confirms wave nature of the light but beyond that it could not establish whether the oscillations in light is transverse or longitudinal. Polarization of light brings forth more information about the wave nature of lights. It also confirms that the light is a transverse wave. In transverse wave, electric field ($\vec{E}$) and magnetic field ($\vec{H}$) vectors oscillate in plane perpendicular to the direction of propagations of wave as shown in Fig. 1.

![Fig. 1](image1)

The oscillation of $\vec{E}$ and $\vec{H}$ vectors can take all the possible directions in the plane perpendicular to the direction of propagation, and such light are called *un-polarized light*. The light is emitted from different atoms which are independent and vibrate in all possible directions; hence the direction of oscillation of $\vec{E}$ and $\vec{H}$ vectors in such light is random. If, by some means, the oscillation of $\vec{E}$ and $\vec{H}$ vectors are confined to one direction then such light is called *polarized light* and the phenomenon is known as polarization, as illustrated in Fig.2

![Fig. 2](image2)

*Polarization is the process or phenomenon in which the waves of light or other electromagnetic radiation are restricted to certain directions of vibration, usually specified in terms of the electric field vector.*
Types of Polarization:
Based on the direction of oscillation the polarization can be of following types.

1. **Linearly polarized**: if the oscillation of light is confined to only one direction, than the light is called linearly polarized. Linear polarization can further be classified into two types
   a. **Vertical polarization**: when the oscillation is in the vertical direction
      ![Vertical polarization](image)
      Fig.3
   b. **Horizontal Polarization**: when the oscillation is in the horizontal direction
      ![Horizontal polarization](image)
      Fig.4

2. **Circularly polarized**: In circular polarization, the electric vector of constant amplitude no longer oscillated but rotated while proceeding in the form of helix. This happens due the combination of two perpendicular linearly polarized lights of same amplitude with a phase difference of $\frac{\pi}{2}$
      ![Circular polarization](image)
      Fig.5
Circularly polarized light also classified into two based on their rotation; Left circular for clock-wise rotation and right circular for anti-clockwise rotation.

3. **Elliptical polarization**: Similar to circular polarization, elliptical polarization is also a result of combination of two perpendicular linearly polarized lights with a phase difference of \( \frac{\pi}{2} \) but with different amplitude. Hence, in the resultant polarized light electrical vector rotates elliptically.

![Fig.6](image)

Elliptically polarized light also classified into two based on their rotation; Left circular for clock-wise rotation and right circular for anti-clockwise rotation.

**Production of Polarized light**

Following are some of the method which can produce polarized lights

1. **Selective Absorption**: The electric vector in unpolarized light oscillated in all possible direction. In selective absorption, materials which allow only light with oscillation in only one direction due to their anisotropic nature. These anisotropic materials are used to produce polarized light.

![Fig.7](image)

A good example is *tourmaline* crystal, *aluminoborosilicate* containing \( Al_2O_3, B_2O_3 \) and \( SiO_2 \). Crystals of *quinine sulfate* called *herapathite* also exhibit polarization by
absorption. Most of the commercial polaroid are made up of these crystals embedded in synthetic material. The most common type of synthetic materials is H-sheet (PVC with iodine) and K-sheet.

2. **Polarization by reflection:** In 1811, Brewster observed that beyond certain angle of reflection the reflection light is plane polarized with polarization axis parallel to the plane of reflection. He also derived a simple relationship between angle of polarization ($\theta_p$) and refractive index ($\mu$) of the medium

$$\mu = \tan \theta_p$$

![Fig. 8](image)

3. **Polarization by Scattering:** Just as unpolarized light can be partially polarized by reflecting, it can also be polarized by scattering (also known as Rayleigh scattering; illustrated Fig 9). Since light waves are electromagnetic (EM) waves (and EM waves are transverse waves) they will vibrate the electrons of air molecules perpendicular to the direction in which they are traveling. The electrons then produce radiation (acting like small antennae) that is polarized perpendicular to the direction of the ray. The light parallel to the original ray has no polarization. The light perpendicular to the original ray is completely polarized. In all other directions, the light scattered by air will be partially polarized. (Source: Boundless. “Polarization By Scattering and Reflecting.” Boundless Physics.)

![Fig. 9](image)
4. **Polarization by double refraction**: Some transparent crystals, such as calcite (CaCO$_3$) and quartz (SiO$_2$), have the property that when one views an object through them one sees two images of the object.

If a narrow beam of light is passed through them, the refracted beam is split into two parts which travel through the crystal and emerge as two separate beams. Such phenomenon is known as *double refraction/birefringence* and such materials are known as **Birefringent material**. One of the beams obeys the ordinary laws of refraction and is called the **ordinary ray (o-ray)**. The other beam is called the **extraordinary ray (e-ray)**. The extraordinary ray does not always lie in the plane of incidence. Its speed, and consequently its index of refraction, depends on its direction of propagation through the crystal. If two beams are analyzed with a Polaroid analyzer one discovers that the two beams are both **polarized**, but that the directions of their vibrations are at right angles to each other.

**Properties of O-Ray & E-ray**

<table>
<thead>
<tr>
<th>Ordinary ray (O-ray)</th>
<th>Extraordinary Ray (E-ray)</th>
</tr>
</thead>
<tbody>
<tr>
<td>O-ray obey ordinary laws of refraction</td>
<td>E-ray does not obey ordinary laws of refraction</td>
</tr>
<tr>
<td>O-ray Horizontally plane polarized and having vibration perpendicular to the principal section</td>
<td>E-ray Vertically plane polarized and having vibration in the principal section</td>
</tr>
<tr>
<td>O-ray travel with same velocity in all directions</td>
<td>E-ray travel with different velocity in different directions</td>
</tr>
</tbody>
</table>

Though e-ray and o-ray has different velocity different direction, but there will be at least one direction in which both e-ray and o-ray will have same velocity which is known as **optical axis**. Principal section is an imaginary section which contains the **optical axis**. If there is only one optical axis, then the crystal is called as **uniaxial crystal**.

These birefringent crystals are classified into **positive** and **negative crystals** based on the speed of the e-ray with respect to o-ray. If velocity ($v_e$) of e-ray is less than the velocity ($v_o$) o-ray then the crystal is known as **positive crystal** and if the velocity ($v_e$) of e-ray is greater than the velocity ($v_o$) o-ray then the crystal is known as **negative crystal**. As the velocity of e-ray and o-
ray are different, hence refractive index corresponding to these rays will also be different. Refractive index corresponding to o-ray is known as ordinary refractive index ($\mu_o$) and refractive index corresponding to e-ray is known as extraordinary refractive index ($\mu_e$).

**Difference between positive and negative crystal**

<table>
<thead>
<tr>
<th>Properties/Crystal</th>
<th>Negative crystal</th>
<th>Positive crystal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Velocity of o-ray and e-ray</td>
<td>$V_o &lt; V_e$</td>
<td>$V_o &gt; V_e$</td>
</tr>
<tr>
<td>Refractive index</td>
<td>$\mu_o &gt; \mu_e$</td>
<td>$\mu_o &lt; \mu_e$</td>
</tr>
<tr>
<td>Example</td>
<td>Calcite (CaCO$_3$)</td>
<td>Quartz (SiO$_2$)</td>
</tr>
</tbody>
</table>

Wave front representation

**Retarders:**

In the previous section, we have seen that positive or negative crystals split the light into e-ray and o-ray which are plane polarized and the direction of polarization for these rays are perpendicular to each other. These rays also travel with different velocities inside the crystal (except when they travel along optical axis) hence there will be a path difference between them as shown in Fig. 11.

Let us consider, plane polarized light incidents on a positive crystal of thickness ‘t’ in a direction perpendicular to the optical axis. It splits into e-ray and o-ray with refractive indexes $\mu_e$ and $\mu_o$. The optical path travelled by these rays will be

- $\mu_et$ for e-ray
- $\mu.ot$ for o-ray.
Path difference between these ray = $(\mu_e - \mu_o)t$
Hence we can control the path difference between the e-ray and o-ray emerging from the positive crystal by controlling the thickness of the crystal ‘t’ because $\mu_e$ and $\mu_o$ are materials property.

Let for some thickness ‘t’ following condition satisfies

$$(\mu_e - \mu_o)t = \frac{\lambda}{2}$$

Path difference between e-ray and o-ray is $\frac{\lambda}{2}$, hence such crystals are called **Half Wave Plates** (HWP) and the corresponding phase difference will be $\pi$

$$(\mu_e - \mu_o)t = \frac{\lambda}{4}$$

And when the above condition satisfies such crystals are called **Quarter Wave Plates** (QWP) and the corresponding phase difference will be $\pi/2$

**Structure of calcite:**

Double refraction occurs in all crystals except those displaying cubic symmetry. If the arrangement of atoms in the calcite crystal is examined in a plane perpendicular to the optical axis, the atoms are found to be symmetrically distributed. If one examines them for any other plane, this is not the case. Both the optical and electrical properties are found to vary in different directions in the crystal.

![Fig.12](image)

It’s a transparent material whose chemical formula is CaCO3. It belongs to rhombohedral class of hexagonal system. A and G are blunt corners with $\angle{BAD} = 120^\circ$ and $\angle{ABC} = 71^\circ$. An imaginary line passing through A and G, which is perpendicular to AB, AD and AE side is optical axis. The ordinary refractive index ($\mu_o$) for calcite crystal is 1.6584 and extraordinary refractive index ($\mu_e$) varies from 1.4864-1.6548 depending on its direction of propagation with respect to optical axis.
Nicol Prism:
Nicol prism is an optical device which is used for producing and analyzing polarized light. Nicol prism was invented by William Nicol in 1828. Calcite crystal is modified such that it eliminates one of the refracted rays by total internal reflection.

Principle: Nicol Prism is based upon phenomenon of double refraction.

Construction: It is constructed from the calcite crystal PQRS having length three times of its width. Its end faces PQ and RS are cut such that the angles in the principal section become 68° and 112° in place of 71° and 109°. The crystal is then cut diagonally into two parts. The surfaces of these parts are grinded to make optically flat and then these are polished. Thus polished surfaces are connected together with special cement known as Canada Balsam.

Working: When a beam of unpolarized light is incident on the face P′Q, it gets split into two refracted rays, named O-ray and E-ray. These two rays are plane polarized rays, whose vibrations are at right angles to each other. The refractive index of Canada Balsam cement being 1.55 lies between those of ordinary (1.6585) and extraordinary (1.4864). It is clear from the above discussion that Canada Balsam layer acts as an optically rarer medium for the ordinary ray and it acts as an optically denser medium for the extraordinary ray. When ordinary ray of light travels in the calcite crystal and enters the Canada balsam cement layer, it passes from denser to rarer medium. Moreover, the angle of incidence is greater than the critical angle, the incident ray is totally internally reflected from the crystal and only extraordinary ray is transmitted through the prism. Therefore, fully plane polarized wave is generated with the help of Nicol prism.

Application as Polarizer and Analyzer:

In order to produce and analyze the plane polarized light, we arrange two Nicol prisms. When a beam of unpolarized light is incident on the Nicol prism, emergent beam from the prism is obtained as plane polarized, and which has vibrations parallel to the principal section. This prism is therefore known as polarizer. If this polarized beam falls on another parallel Nicol prism P2, whose principal section is parallel to that of P1, then the incident beam will behave as E-ray inside the Nicol prism P2 and gets completely transmitted through it. This way the intensity of
emergent light will be maximum. In order to produce and analyze the plane polarized light, we arrange two Nicol prisms.

Fig. 14
Now the Nicol prism P2 is rotated about its axis, then we note that the intensity of merging light decreases and becomes zero at 90° rotation of the second prism (Fig. 13 b). In this position, the vibrations of E-ray become perpendicular to the principal section of the analyzer (Nicol prism P2). Hence, this ray behaves as O-ray for prism P2 and it is totally internally reflected by Canada balsam layer. This fact can be used for detecting the plane polarized light and the Nicol prism P2 acts as an analyzer. If the Nicol prism P2 is further rotated about its axis, the intensity of the light emerging from it increases and becomes maximum for the position when principal section of P2 is again parallel to that of P1 (Fig. 13 c). Hence, the Nicol prisms P1 and P2 act as polarizer and analyzer, respectively.

Malus Law:
According to Malus, when completely plane polarized light is incident on the analyzer, the intensity I of the light transmitted by the analyzer is directly proportional to the square of the cosine of angle between the transmission axes of the analyzer and the polarizer.

Suppose the angle between the transmission axes of the analyzer and the polarizer is θ. The completely plane polarized light form the polarizer is incident on the analyzer. If $E_o$ is the amplitude of the electric vector transmitted by the polarizer, then intensity $I_o$ of the light incident on the analyzer is
\[ I_o = E_o^2 \]

The electric field vector \( E_0 \) can be resolved into two rectangular components i.e \( E_0 \cos\theta \) and \( E_0 \sin\theta \). The analyzer will transmit only the component (i.e. \( E_0 \cos\theta \)) which is parallel to its transmission axis. However, the component \( E_0 \sin\theta \) will be absorbed by the analyzer. Therefore, the intensity \( I \) of light transmitted by the analyzer is,

\[ I = (E_0 \cos\theta)^2 \]

\[ I / I_o = (E_0 \cos\theta)^2 / E_o^2 = \cos^2\theta \]

\[ \therefore I = I_o \cos^2\theta \]

Therefore,

\[ I \propto \cos^2\theta. \]

This proves Malus Law.

**Production and detection of polarized light:**

Let us, theoretically derive an expression for generation of polarized light. As we have seen in the previous section, whenever unpolarized light passed through a polarizer is becomes plane polarized light. And whenever plane polarized light passed through birefringent materials it further split into two perpendicular plane polarized light.

![Fig. 16](image-url)

In Fig.16, unpolarized light falls on polarizer and becomes plane polarized with amplitude ‘\( A \)’ and this plane polarized light fall on a birefringent crystal. Let the plane of polarizations make and angle \( \theta \) with respect to the optical axis of the birefringent crystal. Birefringent materials will split the plane polarized light into two perpendicular plane polarized light as shown in Fig.16.

Now from the Fig.16 it is clear that

Amplitude of the e-ray along O’E will be \( a = A \cos \theta \)

Amplitude of the o-ray along O’O will be \( b = A \sin \theta \)

The wave equation for these rays as they emerge from the birefringent crystal can be expressed in the following equation

\[ X = asin(\omega t+\phi) \] for e-ray
\[ \Rightarrow \frac{x}{a} = \sin(\omega t + \phi) \quad \text{-------(1)} \]

Where \( \phi \) is the phase difference due to the path difference between the rays

\[ Y = b \sin \omega t \] for o-ray

\[ \Rightarrow \frac{Y}{b} = \sin \omega t \quad \text{-------(2)} \]

Expanding equ(1) we get

\[ \frac{x}{a} = \sin (\omega t + \phi) = \sin \omega t \cos \phi + \cos \omega t \sin \phi \quad \text{---------(3)} \]

From equ(2)

\[ \sin \omega t = \frac{Y}{b} \]  
\[ \Rightarrow \cos \omega t = \sqrt{1 - \left(\frac{Y}{b}\right)^2} \]

substituting these values in equ(3) we get

\[ \frac{x}{a} = \frac{Y}{b} \cos \phi + \sqrt{1 - \left(\frac{Y}{b}\right)^2} \sin \phi \]

\[ \Rightarrow \frac{x}{a} - \frac{Y}{b} \cos \phi = \sqrt{1 - \left(\frac{Y}{b}\right)^2} \sin \phi \]

Squaring both side we get

\[ \Rightarrow \left( \frac{x}{a} - \frac{Y}{b} \cos \phi \right)^2 = \left(1 - \left(\frac{Y}{b}\right)^2\right) \sin^2 \phi \]

\[ \Rightarrow \frac{x^2}{a^2} + \frac{Y^2}{b^2} (\sin^2 \phi + \cos^2 \phi) - \frac{2XY}{ab} \cos \phi = \sin^2 \phi \]

\[ \Rightarrow \frac{x^2}{a^2} + \frac{Y^2}{b^2} - \frac{2XY}{ab} \cos \phi = \sin^2 \phi \quad \text{---------(4)} \]

The above equation (equ(4)) is the general form of the light coming out of birefringent material.

**Case-I**

If the phase difference (\( \phi \)) is zero

Then \( \cos \phi = 1 \) and \( \sin \phi = 0 \)
Hence equ(4) reduces to

\[ \Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab} = 0 \]

\[ \Rightarrow \left( \frac{x}{a} - \frac{y}{b} \right)^2 = 0 \]

\[ \Rightarrow \ y = \frac{xb}{a} , \]

This is an equation for a straight line hence the output must be linearly polarized light. Phase difference (\( \phi \)) equal to zero means the thickness of birefringent material is zero. Hence to produce linear/plane polarized light we just need a polarizer.

Case-II

If the phase difference (\( \phi \)) is \( \pi/2 \)

Then \( \cos \phi = 0 \) and \( \sin \phi = 1 \)
And now the equ(4) reduces to

\[ \Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad ----- (5) \]

The above equation is an equation for ellipse. Hence the output is elliptically polarized light. Phase difference of \( \pi/2 \) can be produced by allowing plane polarized light to pass through a quarter wave plate (QWP).

Case-III

In case-II, we have seen that Phase difference of \( \pi/2 \) produces elliptically polarized light. If a and b are equal than the equ(5) will be
$$\Rightarrow X^2 + Y^2 = a^2 \quad ---- \quad (6)$$

for $a = b$

$$\Rightarrow A \sin \theta = A \cos \theta$$

$$\Rightarrow \theta = 45^\circ$$

Equ (6) is an equation for a circle; hence the resultant output is circularly polarized light. Circularly polarized light can be produced by allowing plane polarized light to pass through quarter wave plate (QWP), which is at an angle of $45^\circ$ with respect to the plane of polarization.

Detection of polarized light:

The detection procedure is described in the following flowchart.